Boosting Systematic Risks with CDOs

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Abstract

Structured credit instruments play a significant role at the roots of the credit crisis. It seems as if both practitioners as well as academics have largely disregarded the specific risk profile of portfolio credit derivatives like CDOs, especially their extremely high sensitivity to systematic risks. In this article, we aim at quantifying the systematic risk portion of CDOs based on several risk measures. One of them is a bond representation which admits straightforward risk comparisons of CDOs and conventional bonds. Based on that, we examine common pooling and structuring patterns and find that they are mostly boosting systematic risk. Finally, we shortly address the diversification myth associated with CDOs and show that although being name diversified CDOs imply high factor concentration risk.

Keywords: CDO, Risk Measures, Systematic Risk, Arbitrage, Diversification.

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1 Introduction

The burst of the US housing bubble has triggered an unprecedented depreciation of the whole asset class of credit securitizations. In several waves the major rating agencies downgraded loads of asset backed securities (ABS) and collateralized debt obligations (CDOs). One major problem was that even best rated AAA securities were downgraded, sometimes multiple notches (see Moody's Investors Service [2008]). This rating transition behavior is totally uncommon with conventional bonds and rating agencies always pointed out the historical stability of ratings for structured finance products (see Moody's Investors Service [2007], Fitch Ratings [2006]). A legitimate ensuing question was whether structured finance products like CDOs are intrinsically dangerous or whether they just differ from defaultable bonds. This pressing question was partially answered by credit rating agencies who emphasized that CDO ratings do not have the same meaning as classical bond ratings. But what exactly is the difference? Both are defaultable fixed income securities but they remarkably differ in terms of their risk profile.

A consideration of what CDOs actually are immediately reveals the difference. A CDO investment is a bet that the default rates in the underlying pool will exceed the subordination (which is just a safety buffer) with a certain probability. Now, the type of default clustering which is necessary for tranches with higher subordination to be hit is only possible through systematic effects. CDO tranche risks are primarily driven by systematic risks while conventional bonds are driven by both idiosyncratic as well as systematic components. Although meanwhile this fact is frequently stated in the literature [e.g. Duffie, 2008, Fender, Tarashev, and Zhu, 2008], there are only few more detailed analyses available so far. Specifically, only little knowledge does exist on how certain pooling and structuring patterns imply an increase of systematic risks, such as the choice of collateral or the structure of issued tranches. This issue was largely disregarded because of a business model called "arbitrage CDO". Arbitrage CDOs are vehicles earning money by relatively cheap hedging of a pool of credit risk investments. As we argue below, this "excess spread" may have been increased by arbitrage of systematic risks.

Coval, Jurek, and Stafford [2009a], Coval, Jurek, and Stafford [2009b] and Brennan, Hein, and Poon [2009] analyse aspects of a possible mispricing of CDO tranches, caused by their increased sensitivity to systematic risks. The authors argue that the price investors pay for structured instruments is too high if their investment decision solely relies on the rating. According to Coval, Jurek, and Stafford [2009a] this is especially true for senior tranches which default just when the economy is in a very bad state. In contrast, the results obtained by Brennan, Hein, and Poon [2009] indicate that the AAA-rated tranches are only marginally mispriced, and that the highest profits can be gained with junior tranches. Eckner [2008] comes to similar conclusions.

In this paper we try to quantify CDO sensitivity to systematic risks. As mentioned in Hamerle, Jobst, and Schropp [2008] and Krahnen and Wilde [2009], the risk properties of ABS and CDOs differ significantly from those of corporate bonds. We base our analyses on conditional expected loss (i.e. expected loss conditional upon the market factor). Moving from benign to adverse states of the market factor this measure clearly reveals that tranches have a much higher sensitivity with respect to systematic factors than bonds with a comparable rating. We estimate the asset correlation associated with a CDO tranche treating the structured instrument as a single-name credit instrument (i.e. a loan equivalent). Yahalom, Levy, and Kaplin [2008] from Moody's point out, that this tractable approach "requires appropriate parametrization to achieve a reasonable description of the cross correlation between the structured instrument and the rest of the portfolio." They provide an approach different to ours. For the determination of the correlation parameter they estimate the joint default correlation of two CDOs loading on the same factor and back out the asset correlation that is consistent with this default correlation in the Gaussian single risk factor model. The appoach requires loss given default to be modelled separately.

In our article we examine the systematic risk arising from typical pooling and tranching patterns. We provide a comprehensive default risk assessment of structured credit products and show that first moments are insufficient to reflect risk concentrations and systematic risk sensitivity. To measure the systematic risk and to compare different fixed income products we explain how to calibrate the Gaussian single risk factor model to replicate the risk profile of a tranche. It turns out that the resulting asset correlations of loan equivalents of multi-name derivatives are substantially higher than those of single-name instruments. Furthermore, as mentioned above, buying assets with lower systematic risk and funding them by issuance of assets with higher systematic risk offers arbitrage gains. We show how arrangers may utilize pooling and tranching to increase arbitrage gains.

The rest of the article is organized as follows. In the next section we introduce our model foundation. The section that follows presents classical risk measures for the center and tails of a loss distribution. After that, in Section 4, we describe the aforementioned bond representation of CDO tranches. In Section 5, we examine common pooling and structuring practices against the background of systematic risk. Section 6 shortly discusses the role of diversification with CDO investments. Section 7 concludes.

2 Setup

The following section focuses on presenting the model setup that is used for analyzing the risk characteristics of CDOs.

2.1 CDO Model

A CDO is structured like a balance sheet. In a typical cash CDO a pool of assets (collateral pool) is funded by issuance of debt securities (tranches). In a synthetic transaction a pool of long credit risk protection is hedged by issuance of short credit risk tranches. We shall disregard any principal cash flows here and focus on synthetic transactions. Note that this does not imply any restrictions for our results. We map the collateral pool of defaultable names by means of the de facto standard model in practice, the Gaussian single risk factor model.

Consider a portfolio comprising $i = 1, \dots, N$ credit risky names. Each reference position is represented by the terminal asset value R_i which is constructed as

$$R_i = \sqrt{\rho} \cdot M + \sqrt{1-\rho} \cdot \epsilon_i. \tag{1}$$

M represents the market factor common to all obligors while ϵ_i is an individual innovation. The parameter ρ is usually called asset correlation. It controls the relative relevance of the two factors and thus turns out to be the correlation of any pair $(R_i, R_{i'}), i \neq i'$. ρ may thus be interpreted as the degree of dependence of R_i on M. All names depend on M by the same level of magnitude, i.e. $\rho_i = \rho$ for all i. The factors M and ϵ_i are standard normal distributed and independent. Thus, R_i is also standard normal distributed.

The default of obligor i is modeled as a threshold event, i.e. when R_i falls short of a specific threshold c_i the borrower is in default. This is modeled using the default indicator

$$D_i := \mathbf{1}_{\{R_i \leq c_i\}}, \tag{2}$$

where D_i is unity if $R_i \leq c_i$ and otherwise zero. For our later analyses it is useful to note that ρ is a measure of dependence of R_i on M. In connection with the threshold model its meaning is a little bit different. With respect to D_i we may interpret ρ as a measure of sensitivity of D_i concerning M. This notion will be relevant in later sections.

Now, given the unconditional probability of default $\lambda_i := \mathbb{P}[D_i = 1] = \mathbb{P}[R_i \leq c_i]$ this threshold can be backed out. So

$$c_i = \Phi^{-1}(\lambda_i), \tag{3}$$

where $\Phi(.)$ denotes the standard normal cumulative distribution function.

The structure of R_i establishes independence if conditioned on M. This may be useful in computational terms. For example, given M, the portfolio default rate is an N-fold convolution of independent random variables with (conditional) probabilities of default

$$\lambda_i(M) = \mathbb{P}\left(R_i \leq c_i \mid M\right) = \Phi\left(\frac{c_i - \sqrt{\rho} \cdot M}{\sqrt{1-\rho}}\right). \tag{4}$$

This conditional PD will be relevant below. Note that as with D_i, ρ also measures the degree of sensitivity of $\lambda(M)$ concerning M.

To obtain losses L from default counts or rates we additionally need information regarding the loss given default (LGD) or equivalently the recovery rate $\text{RR}_i = 1 - \text{LGD}_i$. We define LGD_i as the fractional loss of obligor i's exposure at default EAD_i. It is assumed non-stochastic.

Based on this, the loss rate in the collateral pool can be calculated as

$$L = \sum_{i=1}^{N} w_i \cdot LGD_i \cdot D_i \tag{5}$$

where exposure weights are given by $w_i = \frac{EAD_i}{EAD}$ with total exposure $EAD = \sum_{i=1}^{N} EAD_i$.

2.2 Base Case CDO

Next, we introduce a base case CDO configuration.

Collateral Pool The asset side of the base case CDO has a maturity of $T = 5$ years. It comprises $N = 100$ corporate bonds rated BBB ($\lambda_i = \lambda = 3.25\%$ for all i) with equal notional weight $w_i = w = 1/100$. As assumed above, all names have uniform factor dependency $\rho_i = \rho = 10\%$. The recovery rate is also assumed homogeneous among obligors and set to a common level of $\text{RR}_i = \text{RR} = 40\%$ so that loss given default $\text{LGD} = 60\%$. The following table summarizes this configuration.

Parameter	T	N	λ	ρ	w	RR	LGD
Level	5	100	3.25%	10%	1/100	40%	60%

Table 1: Asset pool configuration.

Liability Structure The liability side of the transaction is structured into four tranches. Each tranche (tr) is specified by attachment point a and detachment point b, respectively, with $0 \leq a < b \leq 1$. Table 2 shows the details.

No.	Tranche	a	b	Rating
1	Equity	0%	4%	-
2	Junior Mezzanine	4%	6.5%	B
3	Senior Mezzanine	6.5%	11.5%	BBB
4	Senior	11.5%	100%	AAA
5	Pool	0%	100%	

Table 2: Structure of liabilities.

The structure presented in Table 2 is based on the hitting probabilities of the tranches (for a definition see Section 3.1.1). Ratings of tranches can be related to hitting probabilities as ratings of corporate bonds are related to default probabilities.

3 Risk Measures for CDOs

Our major interest in this article revolves around measuring the objective (i.e. real-world) risks of CDOs. In this section we analyze first and second order measures such as expected and unexpected loss. As descriptive statistics always represent just a summary of certain distributional characteristics we have to consider several measures to get a full picture. In the next three subsections we define these risk measures and then apply them to our sample CDO in subsection four.

3.1 Rating-Based Risk Measures for CDOs

We start with the most relevant first order measures, probability of default (PD) and expected loss (EL). These are the foundations of agency ratings (e.g. of Moody's, Standard and Poor's, and Fitch).

3.1.1 Hitting Probability

The probability of default of a tranche is often referred to as its "hitting probability". It is defined as the probability that a tranche incurs first losses.

$$\lambda^{tr} = \mathbb{P}\left(L > a\right) \tag{6}$$

For example, Standard and Poor's and Fitch determine ratings based on this measure.

3.1.2 Tranche Loss

Given collateral pool loss rate L the loss incurred by a certain tranche $(\mathrm{tr}) = (a,b)$ with $0 \leq a < b \leq 1$ is calculated as

$$L^{\mathrm{tr}} = \frac{1}{b-a} \cdot \left[(L-a) \cdot \mathbf{1}_{\{a<L\leq b\}} + (b-a) \cdot \mathbf{1}_{\{L>b\}} \right]$$
$$= \begin{cases} 0, & L \leq a \\ \frac{L-a}{b-a}, & a < L \leq b \\ 1, & L > b \end{cases} \tag{7}$$

In other words, a tranche (a,b) absorbs only pool losses in excess of a with limit $b-a$. Thus, expected tranche loss is the expectation of the tranche loss rate.

$$\mathbb{E}\left(L^{tr}\right) = \mathbb{P}\left(L > b\right) + \frac{1}{b-a} \cdot \int_a^b \left(l-a\right) dF_L\left(l\right) \tag{8}$$

Moody's determines ratings based on this measure.

3.1.3 Tranche Loss Severity

The loss given default of a tranche is the random variable

$$\mathrm{LGD}^{\mathrm{tr}} = L^{\mathrm{tr}} \mid L > a. \tag{9}$$

Its expectation is a common risk parameter

$$\mathbb{E}\left(\mathrm{LGD}^{\mathrm{tr}}\right) = \mathbb{E}\left(L^{\mathrm{tr}} \mid L > a\right). \tag{10}$$

3.2 Tail Measures

There is a large number of tail or downside risk measures like semivariance, value at risk or expected shortfall. For most of these measures a decomposition is available which admits attributing the contribution of a single constituent to the overall measure.

Value at Risk
The most widely used tail measure is value at risk (VaR). The Value at risk at confidence level $(1-\alpha) \in [0,1]$, $VaR_{1-\alpha}(L)$, is the $(1-\alpha)$-quantile of the loss distribution:

$$VaR_{1-\alpha}\left(L\right) = \min\left\{l \in [0,1] : \mathbb{P}\left(L \leq l\right) \geq 1-\alpha\right\} \tag{11}$$

Losses as high as $VaR_{1-\alpha}(L)$ or higher are defined to occur in $\alpha \cdot 100\%$ of all loss scenarios.

Expected Shortfall
Since VaR has several theoretical deficits the literature prefers expected shortfall $ES_{1-\alpha}(L)$. Expected shortfall at confidence level $(1-\alpha)$ is the expectation of loss above $VaR_{1-\alpha}(L)$. It is given as

$$ES_{1-\alpha}\left(L\right) = \mathbb{E}\left(L | L > VaR_{1-\alpha}\left(L\right)\right). \tag{12}$$

3.3 Analysis of Variance

Our factor model with a common systematic variable M lends itself to analysis of variance and thus admits closer insights into the relevance of systematic and idiosyncratic risk.

$$\mathbb{V}(L) = \mathbb{V}[\mathbb{E}(L \mid M)] + \mathbb{E}[\mathbb{V}(L \mid M)] \tag{13}$$

That is, total risk as measured by variance can be decomposed into the variance of conditional expectations and the expectation of conditional variances. The former is an absolute measure of systematic risk.

The relative importance of the two variance components can be seen by division by $\mathbb{V}(L)$, i.e.

$$\underbrace{\frac{\mathbb{V}[\mathbb{E}(L \mid M)]}{\mathbb{V}(L)}}_{\mathbb{V}^*[\mathbb{E}(L|M)]} + \frac{\mathbb{E}[\mathbb{V}(L \mid M)]}{\mathbb{V}(L)} = 100\%. \tag{14}$$

Below, we only report the first term (the systematic component) as well as the total variance.

3.4 Results

Now we want to apply the risk measures introduced so far to the sample CDO of Section 2.2 and compare the tranches with the whole collateral pool. The latter can be considered as a tranche with attachment points $(a, b) = (0, 1)$. The tranche structure was introduced in Table 2, the collateral pool configuration was shown in Table 1. The results are given in Table 3.

Tranche	1	2	3	4	Pool
a	0	0.04	0.065	0.115	0
b	0.04	0.065	0.115	1	1
λ^{tr}	0.8545250	0.1282040	0.0314680	0.0013850	0.8545250
$\mathbb{E}(L^{tr})$	0.4354460	0.0643830	0.0091760	0.0000310	0.0195130
$VaR_{0.99}(L^{tr})$	1	1	0.38	0	0.084
$ES_{0.99}(L^{tr})$	1	1	0.7213280	0.0220020	0.1045280
$\mathbb{V}(L^{tr})$	0.1087647	0.0448334	0.0051819	0.0000012	0.0003360
$\mathbb{V}[\mathbb{E}(L^{tr} \mid M)]$	0.0655638	0.0245024	0.0027281	0.0000006	0.0002247
$\mathbb{V}^*[\mathbb{E}(L^{tr} \mid M)]$	0.6028040	0.5465210	0.5264680	0.4579700	0.6686360

Table 3: Results: CDO risk measures.

The upper part of the table includes hitting probability, expected loss, VaR, and expected shortfall for each tranche and the whole pool. The lower part contains the systematic part of the variance analysis.

Let us start our interpretation with hitting probability. Hitting probabilities necessarily decrease when we move up the capital structure. The equity tranche incurs losses in 85% of cases and the pool as a whole must necessarily have the same value. The other tranches have a significantly lower first loss risk, e.g., the senior tranche has only 13 basis points. This does not carry over to expected loss. Clearly, the equity tranche always bears the highest level of expected loss while portfolio EL is significantly lower. We see that expected loss decreases as we move up the capital structure. The expected loss of our senior tranche is extremely low ($3.1 \cdot 10^{-5}$).

With respect to tail measures, we observe that VaR and ES are unity for the two lowest tranches which is due to a significant mass singularity at $L^{tr} = 100\%$. In other words, these two tranches have a high "wipe-out" probability. For higher tranches both measures are increasingly smaller. Note that for the senior tranche VaR equals zero since the hitting probability is 0.13% and $0.13\% < 1\% = \alpha$. Here, we recognize a deficit of quantile based tail measures as they may be insensitive to the level of confidence.

To summarize, tranches may have significant tail risk as measured by VaR and ES. Especially for mezzanine tranches there is a large likelihood of total loss. Furthermore, we observe a leverage effect between pool and tranches: junior tranches have significantly higher tail risks in comparison to the whole pool while senior tranches may have clearly lower tail risks. Note, however, that both VaR and ES do not reflect risk concentrations. For example, both the equity tranche as well as the junior mezzanine tranche seem to carry the same tail risk although they differ significantly in terms of expected loss. Similarly, as we will outline below, although both tranches have different sensitivity to the systematic risk factor M, both have the same VaR and ES. Thus, both measures fail to reflect systematic risk sensitivity.

Our last category of risk measures are variance based. Total variance $\mathbb{V}\left[L^{tr}\right]$ as well as systematic variance $\mathbb{V}\left[\mathbb{E}\left[L^{tr} \mid M\right]\right]$ are difficult to interpret in terms of their absolute value. Nevertheless, they are appropriate for risk comparisons, e.g. of different securities with identical expected loss (cf. section 6).

4 Systematic risk characteristics of CDOs

In the last section we have studied several risk measures. We found that the tranches differ clearly in terms of their risk profile both from each other as well as from the pool as a whole. This suggests that bonds and tranches have intrinsically different risk profiles. In this section, we show how to replicate the risk profile of a tranche by means of a standard bond model.

On the one hand, this offers additional insight into the systematic risk characteristics of CDOs. It is often stated that classical bonds and CDO tranches are not comparable and that the rating of the latter may hide important risk components. A bond representation provides an opportunity to compare these two asset classes directly and admits straightforward integration of CDO tranches into a static portfolio model.

4.1 Hitting Probability Profile

In the next two sections we study the default behavior of CDO tranches. First we calculate the conditional hitting probability of a tranche given different values of M. We call this functional dependence a hitting probability profile (HP-profile).

Figure 1 shows conditional hitting probabilities. Red points relate to the mezzanine tranche of our base case and blue points relate to any of the collateral pool bonds. Note that although both have a BBB rating, their risk profiles differ significantly.

Moving from positive (good) factor levels to negative (bad) factor levels (i.e. from right to left) the curve of the tranche increases much faster than that of the bond. A positive economic environment (large values of M) almost completely precludes a tranche hit while for the bond some default risk remains. In such a positive scenario, small changes of M have little impact on

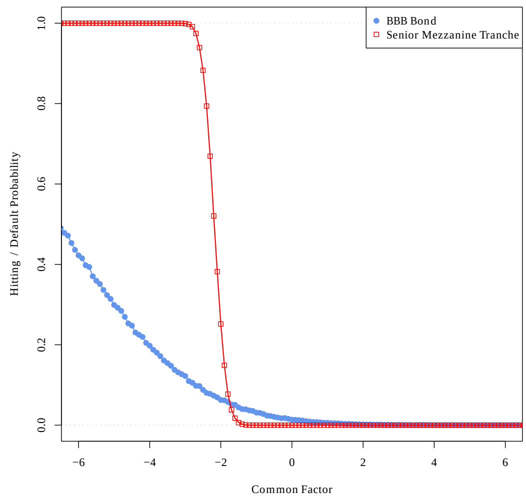


Figure 1: HP-profiles of the BBB mezzanine tranche and a BBB bond.

a tranche's hitting probability but modest influence on a bond's probability of default. Thus, CDOs seem to be more stable against macroeconomic changes.

However, this conclusion is misleading. If the systematic risk factor M falls into a certain critical region (in our example above, this region is at about $M = -2$), CDOs turn out to be extremely sensitive to an economic downturn. Because the steepness of the sensitivity curve is visibly greater in the critical region, even small changes of M may lead to tremendous deterioration of credit quality. In even worse economic conditions a hitting event is almost certain.

Actually, ratings for structured finance assets are thought to be more stable than corporate bond ratings. However, it is also acknowledged that if rating migrations do occur, these changes are of a greater order of magnitude with tranches (see Jobst and de Servigny [2007]). For instance, while rating changes of corporate bonds occur more frequently but only by one or two notches, rating changes of CDO tranches occur seldom but if so they are by multiple notches (see Moody's Investors Service [2008]). In our opinion, the different sensitivity regarding systematic risks can, at least to some extent, explain this phenomenon, that we see, for example, in the current financial crisis.

But the hitting probability profile does not reflect all relevant risk characteristics. The loss given default of a CDO is always a random variable which depends on the systematic risk factor M too. Furthermore, loss distribution and sensitivity to systematic risk depend heavily on the thickness of the tranche. Both is "disregarded" when using a hitting probability profile. An alternative representation comprising these aspects is the expected loss profile $\mathbb{E}\left(L^{\text{tr}} \mid M\right)$.

4.2 Expected Tranche Loss Profile

The expected loss profile (EL-profile) is determined similar to the hitting probability profile. We simulate a number of factor realizations and corresponding losses. Then we collect losses with similar factor realizations and calculate the mean.

Figure 2 shows the EL-profiles of the BBB mezzanine tranche and also that of a BBB bond. Again the differences of both curves are rather obvious. As with HP-profile, the pace of transition from zero to total loss is a measure of sensitivity to systematic risk. Moving from right to

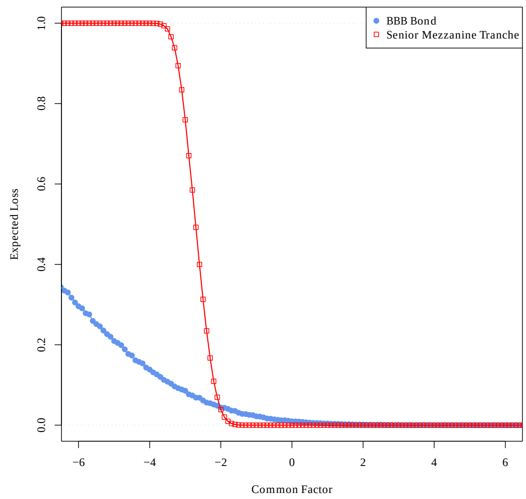


Figure 2: EL-profiles of the BBB mezzanine tranche and a BBB bond.

left the tranche profile rises much more steeply than the corporate bond profile.

Note that in our homogeneous case the expected loss profile of the whole collateral pool is identical to that of any bond in the collateral pool. This means that, in absolute value, the collateral pool and the bond contain exactly the same systematic risk. However, both differ in terms of idiosyncratic risk. While each individual bond still carries a significant amount of idiosyncratic risk the pool as a whole is rather name diversified, i.e. its variance is mainly due to systematic factor movements.

Note that the expected loss profile is also beneficial for pricing considerations. Among others, the price of a (defaultable) security depends on its expected loss as well as on the amount of systematic risk. The EL-profile reflects both. Two credit products with identical EL-profiles are exposed to identical default and systematic risks and therefore should realize the same market prize (see Hamerle, Liebig, and Schropp [2009]).

4.3 Bond Representation

In a "bond representation" the CDO tranche is treated as a single-name credit instrument (i.e. a loan equivalent). The loan equivalent approach requires appropriate parametrization to achieve a reasonable approximation of the tranche's risk profile. We consider the tranche as a "virtual" borrower for which the single-factor model holds. Our first objective is to estimate the "virtual" asset correlation $\hat{\rho}^{\text{tr}}$ of the CDO tranche. Since the tranche's risk profile is given by the EL-profile, our second objective is to approximate the EL-profile of the tranche (obtained via simulation) by the corresponding EL-profile of the "virtual" bond as accurately as possible. In general we have the decomposition

$$\mathbb{E}\left(L^{\text{tr}} \mid M\right) = \lambda^{\text{tr}}(M) \cdot \mathbb{E}\left(LGD^{\text{tr}} \mid M\right) \tag{15}$$

In a first approach the HP-profile $\lambda^{\text{tr}}(M)$ can be used in the bond representation. Considering a CDO tranche as a "virtual" bond, the conditional hitting probability is expressed in the single-factor model as a function of M by

$$\lambda^{\text{tr}}(M) = \Phi\left(\frac{c^{\text{tr}} - \sqrt{\hat{\rho}^{\text{tr}}}\, M}{\sqrt{1-\hat{\rho}^{\text{tr}}}}\right) \tag{16}$$

The default threshold c^{tr} can be calculated using the unconditional hitting probability λ^{tr} given in (6).

The only remaining parameter in (16) is the "virtual" asset correlation $\hat{\rho}^{\text{tr}}$. This parameter can now be determined in a way, that the function assigned in (6) approximates the simulated HP profile as accurately as possible (see Hamerle, Jobst, and Schropp [2008] or Donhauser [2010]).

Another approach stems from the rating agency Moody's (see Yahalom, Levy, and Kaplin [2008]). In this procedure the "virtual" asset correlation $\hat{\rho}^{\text{tr}}$ is calculated by assuming two identical CDO tranches with collateral pools containing different assets having same characteristics concerning number and risks. Furthermore it is assumed that both tranches can be modeled using a single factor model with identical risk parameters. The "virtual" asset correlation can then be determined using the simulated joint default probability and the bivariate normal distribution from the single factor model. Both approaches provide identical results (see Donhauser [2010]).

The main characteristic of both approaches is the goal of finding (the risk parameters of) a "virtual" bond whose conditional hitting probability matches the simulated HP-profile as close as possible. To copy the EL-profile using the approaches above we need the conditional expected loss given default of the tranche (see (15)), which can be calculated via stochastic simulation. An approximation using a common function is not possible without further complexity and the assumtion of a constant expected LGD provides an insufficient approximation of the EL-profile. Then again the bond representation aims to be as basic as possible and comparable to approaches that are used for modeling traditional single-name products.

Therefore we introduce another approach. We start directly from the EL-profile on the left-hand side of (15). Assuming a constant $\widetilde{LGD}^{tr}$ we look for a "virtual" bond whose conditional hitting probability approximates the simulated EL profile as accurately as possible.

Using (15) the "implied" hitting probability of the "virtual" bond conditional on the systematic risk factor is then determined from

$$\tilde{\lambda}^{\text{tr}}(M) = \frac{\mathbb{E}\left(L^{\text{tr}} \mid M\right)}{\widetilde{LGD}^{tr}}. \tag{17}$$

The tranche's "implied" unconditional hitting probability is given by

$$\tilde{\lambda}^{\text{tr}} = \frac{\mathbb{E}\left(L^{\text{tr}}\right)}{\widetilde{LGD}^{tr}}. \tag{18}$$

The unconditional expected tranche loss $\mathbb{E}\left(L^{\text{tr}}\right)$ is also calculated in the course of in the simulation and the default threshold c^{tr} is

$$c^{\text{tr}} = \Phi^{-1}\left(\tilde{\lambda}^{\text{tr}}\right). \tag{19}$$

In the next step, the tranche LGD, $\widetilde{LGD}^{tr}$, is determined as the maximum loss of the tranche.

The maximum loss of the collateral pool is given by

$$L_{max} = \sum_{i=1}^{N} w_i \cdot (1 - \text{RR}_i) \tag{20}$$

which reduces to $L_{max} = 1 - \text{RR} = \text{LGD}$ for a homogenous portfolio.

Based on this, the maximum tranche loss of a tranche with attachment point a and detachment point b is:

$$\widetilde{LGD}^{tr} = \begin{cases} \frac{\min(L_{max}, b) - a}{b - a}, & \text{if } L_{max} > a \\ 0, & \text{else} \end{cases} \tag{21}$$

In general, $\widetilde{LGD}^{tr} = 1$ for all tranches except senior or super-senior tranches, i.e. $\widetilde{LGD}^{tr} < 1$ only for the tranche with the highest seniority ($b = 1$). For the non-senior tranches, the "implied" hitting probability profile of the "virtual" bond is equal to the EL-profile, while the EL-profile is scaled up for the senior tranche with $\widetilde{LGD}^{tr} < 1$.

Finally, the "virtual" asset correlation $\hat{\rho}^{\text{tr}}$ is estimated by means of optimization

$$\arg\min_{\hat{\rho}^{\text{tr}}} \left\{ \sum_{k=1}^{K} \left[\tilde{\lambda}^{\text{tr}}(m_k) - \hat{\lambda}^{\text{tr}}(m_k) \right]^2 \middle| \hat{\rho}^{\text{tr}} \in [0, 1] \right\} \tag{22}$$

where

$$\hat{\lambda}^{\text{tr}}(m_k) = \Phi\left(\frac{\Phi^{-1}\left(\tilde{\lambda}^{\text{tr}}\right) - \sqrt{\hat{\rho}^{\text{tr}}} \cdot m_k}{\sqrt{1 - \hat{\rho}^{\text{tr}}}} \right), \tag{23}$$

$(m_k)_{k=1}^{K}$ is a sufficiently accurate discretization of the support of M and $\tilde{\lambda}^{\text{tr}}(m_k)$ are simulated "implied" conditional hitting probabilities evaluated at m_k.

In summary, a CDO tranche is approximated by a "virtual" bond in a single factor model according to (1), with "virtual" probability of default $\tilde{\lambda}^{\text{tr}}$, "virtual" asset correlation $\hat{\rho}^{\text{tr}}$ and $\widetilde{LGD}^{tr}$ as "virtual" loss given default of the bond representation. The approach ensures that the EL profile of the "virtual" bond resembles that of the simulated EL-profile of the CDO tranche. As outlined above this bond representation based on the EL-profile is a more appropriate approximation of the default behavior and risk profile of the CDO tranche than an approximation based on the conditional hitting probability (with fixed LGD).

4.4 Results

Applying the procedure described in the last subsection to our sample configuration yields the results presented in Table 4.

Tranche	a	b	$\tilde{\lambda}^{\text{tr}}$	$\widetilde{LGD}^{tr}$	$\hat{\rho}^{\text{tr}}$	$\sigma_{\hat{\rho}^{\text{tr}}}$	MSE
Equity	0	0.04	0.435135	1	0.421919	0.005697	0.000231
Junior Mezzanine	0.04	0.065	0.064186	1	0.739621	0.000802	0.000008
Senior Mezzanine	0.065	0.115	0.009139	1	0.753105	0.000520	0.000006
Senior	0.115	1	0.000056	0.548023	0.321983	0.003523	0.002821
Pool	0	1	0.032490	0.6	0.100016	5.90e-06	3.23e-08

Table 4: Approximation results for the bond representation

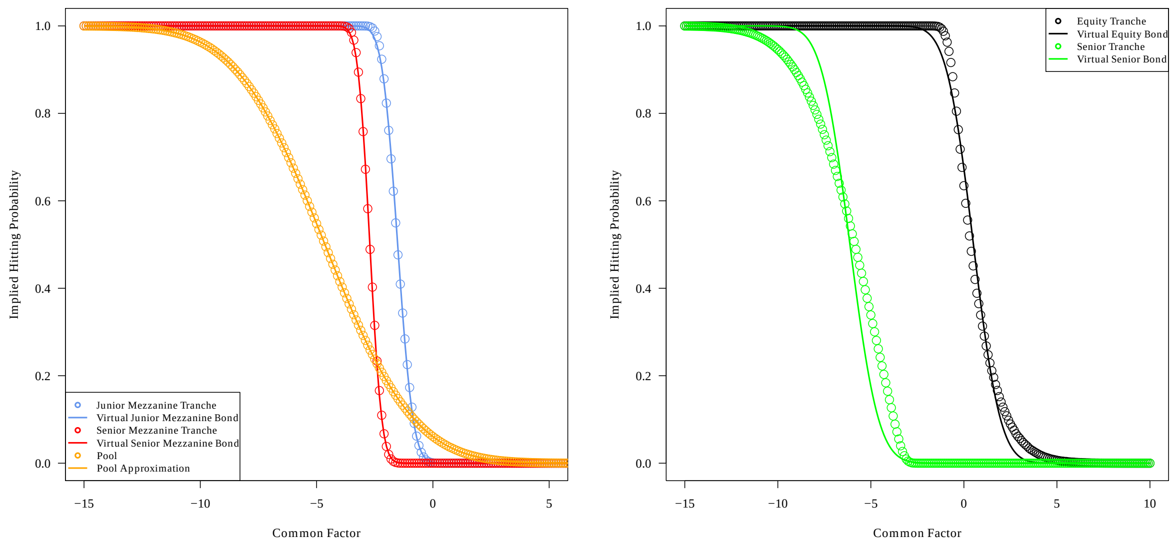
Several points are worth mentioning. First, the difference between $\tilde{\lambda}^{\text{tr}}$ and λ^{tr} as reported in Table 3 derives from the fact that we fit to $\tilde{\lambda}^{\text{tr}}(M)$ instead of $\lambda^{\text{tr}}(M)$. Assuming the constant

$\widetilde{LGD}^{tr}$ implies that the hitting probability of the bond representation has to be lower than the original hitting probability. For instance, the equity tranche has $\lambda^{\text{Equity}} = 0.854$ but $\tilde{\lambda}^{\text{Equity}} = 0.435$. Next, we see that the calibrated asset correlations of all tranches are higher than for the pool as a whole. The highest "correlation leverage" can be observed with mezzanine tranches while equity and senior tranches are more moderate. The reason for the latter is that equity tranches suffer the majority of expected losses (which occur in both good and bad times). On the other end of the capital spectrum, the senior tranche carries the end of the pool loss tail and is thus driven by unexpected losses. Nevertheless, due to its usually large notional share (in our case $100\% - 11.5\% = 88.5\%$) its calibrated $\hat{\rho}^{\text{tr}}$ is lower than that of the mezzanine tranches.

Now, how good is our approximation? To answer this question we measure goodness of fit by means of mean squared error defined by

$$MSE = \frac{1}{K}\sum_{k=1}^{K}\left[\tilde{\lambda}^{\text{tr}}(m_k) - \hat{\lambda}^{\text{tr}}(m_k)\right]^2. \tag{24}$$

The results in Table 4 show that mezzanine tranches can be better approximated than the lowest and highest tranche. This is purely due to a lack of functional flexibility of the Gaussian copula model. Being point symmetric about its inflection point the Gaussian conditional PD model is perfectly able to reproduce the profiles of mezzanine tranches. Because of their position at the ends of the capital structure the equity as well as the senior tranche profile do not fulfill the symmetry criterion. Therefore, the goodness of fit of these tranches is less satisfactory. The pool is particularly well approximated. Figure 3 shows real (i.e. collateral pool simulation based) profiles and fitted profiles. The graphs fully reflect the results of MSE calculation.



(a) Junior Mezzanine Tranche, Senior Mezzanine Tranche and Pool

(b) Equity and Senior Tranche

Figure 3: Goodness of fit of approximated conditional expected loss. Dotted lines: real conditional expected loss, solid lines: fitted conditional expected loss.

5 Structures Boosting Systematic Risks

In the last section we showed the leverage effect of tranching on systematic risk and provided a bond representation of CDO tranches. In this section we examine the consequences on past pooling and structuring practice.

5.1 Systematic Risk and CDO-Pricing

CDOs transform collateral pool loss distributions into new and different loss distributions. Risks are completely reallocated. The hitting probability and expected loss of a large portion of the capital structure are distinctly lower than for an average collateral pool asset. It is known that many investors based their investment decisions mainly on the ratings of the CDO tranches. The ratings rely on assessment either of hitting probabilities or of expected losses due to default. However, measures of default probability or expected loss do not take account of the states of the economy in which the losses occur. But it is also well known that systematic risk is price relevant. Therefore, depending on their exposure to systematic risks, securities with identical credit ratings can trade at very different prices and command a wide range of yield spreads.

In the last section we discussed the leverage effect of tranching on systematic risk and showed that the systematic risk of all tranches rises dramatically. This indicates that investors should receive much higher spreads for their investments in CDOs as a risk premium than that which they are paid on corporate bonds with identical credit ratings. However, this was not the case in the years prior to inception of the financial crisis (see Brennan, Hein, and Poon [2009]). Due to rising demand for structured credit products the spread differences between CDOs and corporate bonds with comparable ratings narrowed. In some cases the spreads of corporate bonds were even higher than the spreads of corresponding tranches. If investors are guided solely by the tranches' ratings in their valuation process and ignore the increased systematic risk, there is a general way to CDO arbitrage. In this case, it is advantageous for the CDO arranger to put together the collateral pool in such a way that the tranches create securities with high systematic risk. If the tranches can be sold at prices comparable to those of corporate bonds with same ratings, a maximum profit potential for the arranger is implied. For details see Hamerle, Liebig, and Schropp [2009].

In the following we describe some possibilities of generating tranches with high systematic risk. It comes as no surprise that precisely these types of transactions can be found in many CDOs issued prior to the outbreak of the financial crisis.

5.2 Collateral Pool Diversification

A frequently stated benefit of CDOs or ABS is the fact that the investment is already diversified (see Fitch Ratings [2008a]). Pooling reduces idiosyncratic loss variance and becomes more effective the larger the pool is and the less heterogeneous the pool constituents are. These are standard results from portfolio theory. Questionable, however, is whether a diversified investment is really a benefit to the investor. Our previous results suggest that the systematic risk of tranches increases as pool diversification increases.

We want to examine this now. To that end, we compare tranche risk measures of the $N = 100$ pool with those of an $N = 1000$ pool as well as with an infinitely fine grained pool ($N \to \infty$). Since increasing the number of names changes PD as well as EL (and thus the rating) of each tranche, we have to modify the capital structure in order to maintain comparability. The idea is that tranche comparisons are only useful when they bear the same level of expected loss. Thus, starting from the highest tranche we change attachment points in a way that each tranche obtains the same expected loss $\mathbb{E}\left(L^{\text{tr}}\right)$ as the corresponding tranche in the base case with $N = 100$. The equity tranche cannot be compared since its attachment point is fixed ($a = 0$). The infinitely fine grained case is known as Vasicek approximation and renders computation of risk measures often significantly easier. Table 5 shows the resulting risk measures.

Tranche	1	2	3	4	Pool
			$N = 1000$		
a	0.000	0.036	0.059	0.101	0.000
b	0.036	0.059	0.101	1.000	1.000
λ^{tr}	0.9984310	0.1274960	0.0272250	0.0017100	0.9984310
$\mathbb{E}\left(L^{tr}\right)$	0.4923910	0.0643760	0.0091550	0.0000310	0.0195030
$VaR_{0.99}\left(L^{tr}\right)$	1	1	0.3604350	0	0.0738000
$ES_{0.99}\left(L^{tr}\right)$	1	1	0.6686230	0.0179260	0.0896710
$\mathbb{V}\left(L^{tr}\right)$	0.0829043	0.0315945	0.0028030	0.0000005	0.0002364
$\mathbb{V}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.0777922	0.0288771	0.0025666	0.0000004	0.0002252
$\mathbb{V}^{*}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.9383375	0.9139929	0.9156945	0.8963110	0.9526432
$\hat{\lambda}^{\text{tr}}$	0.4924453	0.0643278	0.0091739	0.0000559	0.0325076
$\hat{\rho}^{\text{tr}}$	0.4896021	0.9195403	0.8969803	0.3296629	0.0999921
L^{tr}_{max}	1	1	1	0.5551323	0.6000000
			$N \to \infty$		
a	0.000	0.035	0.058	0.099	0.000
b	0.035	0.058	0.099	1.000	1.000
λ^{tr}	1	0.1270540	0.0270495	0.0017655	1
$\mathbb{E}\left(L^{tr}\right)$	0.5001338	0.0643764	0.0091725	0.0000306	0.0195000
$VaR_{0.99}\left(L^{tr}\right)$	1	1	0.3568718	0	0.0726457
$ES_{0.99}\left(L^{tr}\right)$	1	1	0.6843768	0.00330547	0.0890376
$\mathbb{V}\left(L^{tr}\right)$	0.0795542	0.0299154	0.0025724	0.0000004	0.0002247
$\mathbb{V}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.0795542	0.0299154	0.0025724	0.0000004	0.0002247
$\mathbb{V}^{*}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	1	1	1	1	1
$\hat{\lambda}^{\text{tr}}$	0.5001338	0.0643764	0.0091725	0.0000550	0.0325
$\hat{\rho}^{\text{tr}}$	0.4995689	0.9422924	0.9152650	0.3312429	0.1000
$\widetilde{LGD}^{tr}$	1	1	1	0.5559412	0.6000

Table 5: Risk measures for different portfolio sizes: $N = 1000$ and $N \to \infty$.

First, the estimates of $\hat{\rho}^{\text{tr}}$ show a clear increase in comparison with the base case which means that tranche sensitivity is higher for $N = 1000$ and still higher for $N \to \infty$. Our relative variance measure additionaly signals that all tranches have virtually no idiosyncratic risk for $N = 1000$. Almost the whole tranche loss variance is driven by the systematic factor. For $N \to \infty$ the variance is purely systematic. This, however, does not implicate a reduction of the investor's portfolio variance. Instead, it means higher risk concentration. In other words, diversification in the pools underlying a CDO tranche implies concentration risk in the investor's portfolio. We refer to Section 6 as well as Hamerle and Plank [2009] for a more detailed analysis of diversification and risk concentration with CDOs.

5.3 ABS CDOs

In our previous analysis we have shown that CDO tranches bear significant systematic risk. Thus, we may hypothesize that CDOs including tranches in the collateral pool have still higher systematic risk exposure. Such double layer structures are known as CDOs of ABS, ABS CDOs or Structured Finance CDOs (SF CDOs).

We shall investigate this hypothesis now. Exact calculation of the loss distribution and risk measures of ABS CDOs is usually burdensome. Our bond approximation admits a simplified solution. First, given a pool configuration we determine $\hat{\lambda}^{\text{tr}}$ and $\hat{\rho}^{\text{tr}}$ of our tranches. These are the so called "inner" CDOs which form the collateral pool of an "outer" CDO. Thus, our second step is to simulate a portfolio of bonds with $\hat{\lambda}^{\text{tr}}$ and $\hat{\rho}^{\text{tr}}$. Based on this, we determine risk measures for the capital structure of the outer CDO (the ABS CDO tranches).

For our ABS CDO we chose a collateral pool with the following composition as shown in Table 6.

Parameter	Mezzanine RMBS	BBB Bond
Absolute Amount	70	30
Notional Share	70%	30%
Rating	BBB	BBB
PD	0.9139%	3.25%
ρ	75.31%	10%
LGD	100%	60%

Table 6: ABS CDO collateral pool composition.

We do not choose a pure RMBS pool as market practice was to mix bonds and RMBS (see Bank for International Settlements [2008]). The asset pools underlying the RMBS comprise 100 BBB bonds as shown in the right column. Thus, these bond types are used as collateral pool for the inner CDOs and also as 30% of the collateral pool of the outer CDO. However, bonds in the collateral pool of the outer CDO are assumed to be driven by a separate, uncorrelated factor $\tilde{M}$. Bonds in the inner CDOs' collateral pools are driven by M.

As above, the capital structure of the outer CDO is chosen top down so that expected tranche losses of senior and mezzanine tranche equal those of the base case (Table 4). Thus, all tranches except for the equity tranche of ABS CDO and base case CDO are comparable. The structure is given in Table 7.

No.	Tranche	a	b
1	Equity	0%	2%
2	Mezzanine	2%	63.2%
3	Senior	63.2%	100%
4	Pool	0%	100%

Table 7: Outer CDO structure based on expected tranche loss.

We omit a junior mezzanine tranche here. Interestingly, the senior tranche demands much higher subordination to achieve base case levels of expected loss. This in turn suggests extreme tails of the ABS CDO loss distribution. As a result of such high senior subordination the mezzanine tranche is extremely large.

The resulting risk measures are shown in Table 8.

Tranche	Equity	Mezzanine	Senior	Pool
a	0	0.02	0.632	0
b	0.02	0.632	1	1
λ^{tr}	0.5892880	0.1026500	0.0003280	0.5892880
$\mathbb{E}\left(L^{tr}\right)$	0.3332090	0.0090900	0.0000330	0.0122400
$VaR_{0.99}\left(L^{\text{tr}}\right)$	1.0000000	0.2614380	0.0000000	0.1800000
$ES_{0.99}\left(L^{\text{tr}}\right)$	1.0000000	0.5038230	0.1005930	0.3295660
$\mathbb{V}\left(L^{tr}\right)$	0.1173862	0.0038131	0.0000155	0.0016823
$\mathbb{V}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.0581641	0.0036870	0.0000134	0.0016052
$\mathbb{V}^{*}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.4954930	0.9669400	0.8652660	0.9542100
$\hat{\lambda}^{\text{tr}}$	0.3336270	0.0090386	0.0000490	0.0138822
$\widetilde{LGD}^{tr}$	1.0000000	1.0000000	0.6739130	0.8800000
$\hat{\rho}^{\text{tr}}$	0.4661735	0.8271813	0.5554627	0.5547004

Table 8: Risk measures for the ABS CDO.

We see clearly increased bond correlation estimates $\hat{\rho}^{\text{tr}}$. Tail risk measures of the senior tranche and pool are much higher than for the base case. For instance, for the collateral pool ES is 0.10 for the base case and 0.32 for the ABS CDO. For the mezzanine tranche ES and VaR are lower which is certainly due to the comparatively large size. Finally, from $\mathbb{V}^{*}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$ we see that both mezzanine as well as senior tranche are almost exclusively driven by systematic risk.

As recently pointed out by Fitch Ratings [2008b] as well, we may summarize that ABS CDOs imply an even higher level of systematic risk sensitivity. We find this fact in tail measures, bond approximation parameters, as well as variance measures.

5.4 Thin Tranches

As we saw in the last sections, systematic risk on the asset side can be increased by (1) increasing the pool size and (2) choosing assets with higher systematic risk. In this subsection we turn to the liability side.

Donhauser [2010] shows the effect of subordination and tranche width on their sensitivity to systematic risks in detail. In summary the subordination ”only” affects a tranche’s probability to be hit by losses. The longer the tranche is protected against losses in the collateral pool (the higher the subordination of the tranche is), the smaller the resulting hitting probability will be. Reducing the attachment point without changing the tranche width shifts the EL-profile towards better (more positive) realizations of the systematic risk factor. Thereby the steepness of the profile is almost unaffected. In contrast to the degree of subordination, the tranche width does affect the systematic factor sensitivity. The smaller the tranche width is, the bigger becomes the contribution of a single hitting event to the tranche loss rate. So the slope of the profile increases with diminishing tranche width.

In practice senior tranches are very wide and mezzanine tranches are usually thin. Very thin tranches (so called "tranchelets") became increasingly popular in recent times. As described by Tavakoli [2008] sometimes a thin AAA tranche was cut at the bottom of the senior tranche making the superordinate tranche even more safe. That's why it is called "super senior". We are interested in how these structurings affect the risk measures of the resulting tranches. To that end we split the mezzanine tranche $(a, b) = (0.065, 0.115)$ into five tranchelets with 1% width. Furthermore, we form an additional 5% tranche at the bottom of the senior tranche.

The risk measures of the new tranchelets are shown in table 9.

Parameter	Original Mezzanine Tranche	Mezzanine Tranchelets				
a	0.065	0.065	0.075	0.085	0.095	0.105
b	0.115	0.075	0.085	0.095	0.105	0.115
λ^{tr}	0.0315800	0.0315800	0.0157320	0.0078520	0.0055700	0.0028320
$\mathbb{E}\left(L^{tr}\right)$	0.0091080	0.0210190	0.0120310	0.0066170	0.0037190	0.0021570
$VaR_{0.99}\left(L^{tr}\right)$	0.38	1	0.9	0	0	0
$ES_{0.99}\left(L^{tr}\right)$	0.7222750	1	1	0.8546870	0.6758710	0.7686790
$\mathbb{V}\left(L^{tr}\right)$	0.0051585	0.0183948	0.0106270	0.0060093	0.0033259	0.0019329
$\mathbb{V}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.0027287	0.0083803	0.0046377	0.0024539	0.0013101	0.0007258
$\mathbb{V}^*\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.5289730	0.4555780	0.4364090	0.4083530	0.3939260	0.3755060
$\hat{\lambda}^{\text{tr}}$	0.0092140	0.0212780	0.0121400	0.0067110	0.0037650	0.0021770
$\widetilde{LGD}^{tr}$	1	1	1	1	1	1
$\hat{\rho}^{\text{tr}}$	0.7517390	0.7966750	0.8084510	0.8184150	0.8247900	0.8313230

Table 9: Risk measures for thin mezzanine tranches.

Table 9 and table 10 show the risk measures for the original tranches as well as the new (thinner) tranches.

Most importantly, from $\hat{\rho}^{\text{tr}}$ we see that the systematic sensitivity of the new tranches is significantly higher. For the mezzanine tranches it rises from 0.75 for the original mezzanine tranche to 0.8 on average for the tranchelets. The same holds true for the senior tranches where the original senior tranche has $\hat{\rho}^{\text{tr}} = 0.32$ while the new senior tranche shows an asset correlation of $\hat{\rho}^{\text{tr}} = 0.81$. A similar result is reflected by our relative variance measure.

To summarize, we find that systematic risk factor sensitivity decreases with tranche width. This represents potential for a wider funding gap.

6 Diversification and Concentration of Risk

So far, we found that CDO tranches generally carry high systematic risk. We hypothesized that although the underlying pool of a CDO may be highly diversified, this does not apply to the

Parameter	Original Senior Tranche	New Senior Tranche	Super Senior Tranche
a	0.115	0.115	0.165
b	1	0.165	1
λ^{tr}	0.0013810	0.0013810	0.0000940
$\mathbb{E}\left(L^{tr}\right)$	0.0000310	0.0005160	0.0000020
$VaR_{0.99}\left(L^{tr}\right)$	0.0000000	0.0000000	0.0000000
$ES_{0.99}\left(L^{tr}\right)$	0.0226460	0.3721510	0.0252260
$\mathbb{V}\left(L^{tr}\right)$	0.0000012	0.0002974	0.0000001
$\mathbb{V}\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.0000006	0.0001264	$<$0.0000001
$\mathbb{V}^*\left[\mathbb{E}\left(L^{tr} \mid M\right)\right]$	0.4579000	0.4251280	0.3825020
$\hat{\lambda}^{\mathrm{tr}}$	0.0000570	0.0005140	0.0000050
$\hat{\rho}^{\mathrm{tr}}$	0.3209300	0.8083430	0.3867380
$\widetilde{LGD}^{tr}$	0.5480230	1.0000000	0.5209580

Table 10: Risk measures for thin senior tranche and super senior tranche.

investor portfolio the CDO is part of. Indications for this proposition were seen in the high asset correlation of the CDO's bond representation.

We want to elaborate on this in more depth. To that end, we compare four different homogenous investment portfolios containing 100 respectively 200 similar securities. A pure bond portfolio, a pool of pro-rata bond-portfolio investments, a portfolio of CDO tranches and a portfolio of ABS CDO tranches. We do that for two different levels of expected loss, each linked to a rating of either BBB or AAA. The attachment and detachment points are set to match the desired level expected loss. Note that the first four alternatives have equal expected losses as do the second four alternatives. The CDOs are backed by BBB bonds and the ABS CDOs are backed by a pool of mezzanine CDO tranches. Both CDOs and ABS CDOs are modelled as "loan-equivalents". Altogether, we compare eight different investment alternatives. In the following all the investment alternatives are described in detail.

1. The first portfolio comprises BBB-rated corporate bonds. As in the sections before, this goes along with a probability of default of 3.25%, a loss severity that is set to $LGD = 60\%$ and an assumed asset correlation of $\rho = 10\%$. Thus the expected loss is $\mathbb{E}(L) = 1.95\%$.

2. The second case is an pro rata investment into a pool of homogenous bond-portfolios. Each portfolio comprises 100 corporate bonds with risk parameters as described in alternative 1. So the expected loss is $\mathbb{E}(L) = 1.95\%$ too.

3. Alternative number three is an investment into a pool of mezzanine CDO tranches with a=6% and b=8.6%. The collateral pool the tranche is related to, is identically composed to alternative 1. The tranches are modelled as loan-equivalents with parameters being calibrated as shown in section 4.3. This leads to a virtual asset correlation of $\hat{\rho}^{\mathrm{tr}} = 77.59\%$. By choosing the attachment point a=6% this alternative has a virtual default probability of 1.95%. In connection with the detachment point b=8.6% and the virtual loss given default of 100% the expected loss equals 1.95% as well.

4. The investment alternative number four is an investment into a pool of BBB-rated ABS-CDO tranches. Each of these second layer tranches is backed by 100 mezzanine CDO tranches as presented in alternative 3. For this case we find a virtual asset correlation of $\hat{\rho}^{\text{tr}} = 97.57\%$. To achieve expected loss neutrality we choose a=17.5% which provides an implied hitting probability of $\tilde{\lambda}^{\text{tr}} = 1.95\%$. With b=46.7% and $\widetilde{LGD}^{tr} = 100\%$ the expected loss is once more set to $\mathbb{E}(L) = 1.95\%$.

5. The portfolio of investment alternative number five comprises AAA-rated corporate bonds. This is linked to a default probability of 0.15%. The loss given default is set to $LGD = 60\%$ and the asset correlation is assumed to be $\rho = 10\%$. Thus the resulting expected loss of holding this portfolio is $\mathbb{E}(L) = 0.09\%$.

6. Equally as for alternatives 1 and 2, in the sixth case we consider an pro rata investment into a pool of homogenous bond-portfolios (each including 100 corporate bond positions), that are equally composed to those presented in alternative 5. So the expected loss is $\mathbb{E}(L) = 0.09\%$ too.

7. The portfolio of investment alternative seven consists of AAA-rated senior CDO tranches. The tranches are linked to the collateral pool introduced in case 3 of this list. The implied parameters for simulating this investment are $a = 5.52\%, b = 100\%, \hat{\rho}^{\text{tr}} = 21.34\%, \tilde{\lambda}^{\text{tr}} = 0.156\%$ and $\widetilde{LGD}^{tr} = 57.66\%$. So the expected loss of 0.09% is equal to those of alternatives 5 and 6. Note that the tranches of case 3 and 7 are linked to identical collateral pools. The resulting structures are set just to gain the desired levels of expected loss. The overlap of the two tranches therefore does not bother because we do not look at two tranches of the same capital structure here.

8. The eighth and last alternative is an multi-layer investment into a portfolio of AAA-rated ABS-CDO tranches, each backed by a pool of 100 mezzanine CDO tranches as presented in alternative 3. The expected loss of 0.09% is accomplished by setting $a = 87.1\%, b = 100\%, \hat{\rho}^{\text{tr}} = 95.8\%, \tilde{\lambda}^{\text{tr}} = 0.09\%$ and $\widetilde{LGD}^{tr} = 100\%$.

We aim at comparing alternative 1 to 4 and 5 to 8. The resulting risk measures of the portfolio alternatives can be compared in Table 11. We show the two different portfolio sizes $N = 100$ and $N = 200$.

As the most important result, we find that tail risk measures are signficantly higher for CDO tranche portfolios than for bond pools. For example, VaR of alternative (1) and (2) is about 8% but 49% and 93%, respectively, for alternative (3) and (4). The same holds true for the high grade alternatives (5-8). However, for alternative (8) and $N = 100$ we see again the insensitivity issue as discussed above (i.e. the confidence level is too low). Expected shortfall is more reliable here and shows clearly increased values for the tranche portfolios. Our variance measures agree with these findings. They are consistently higher for tranche portfolios. In addition, relative systematic variance is very high for all portfolios except for the pure bond portfolios (1) and (5). It is important to note that although these portfolios seem highly diversified since $\mathbb{V}^*[\mathbb{E}(L \mid M)]$ is close to unity for alternatives (2), (3), (4) and (6), (7), (8), they bear different levels of systematic risk. From the tail risk measures in connection with the relative variance measure we see that alternatives (2) and (6), the pro-rata bond pool investments, are name diversified as well as with moderate systematic risk, while (3), (4) as well as (7), (8) are name diversified but still carry high systematic risk.

Alternative	1	2	3	4	5	6	7	8
				$N = 100$				
$\mathbb{E}(L)$	1.95%	1.95%	1.95%	1.95%	0.09%	0.09%	0.09%	0.09%
$VaR_{0.99}(L)$	8.4%	7.26%	49%	93%	1.2%	0.56%	1.15%	0%
$ES_{0.99}(L)$	10.52%	8.80%	70.1%	99%	1.98%	0.78%	2.21%	21.69%
$\mathbb{V}(L)$	0.034%	0.022%	0.74%	1.48%	0.0007%	0.0001%	0.0010%	0.056%
$\mathbb{V}[\mathbb{E}(L \mid M)]$	0.022%	0.022%	0.73%	1.48%	0.0001%	0.0001%	0.0005%	0.056%
$\mathbb{V}^*[\mathbb{E}(L \mid M)]$	66.90%	100%	98.41%	99.72%	19.89%	100%	47.19%	99.39%
				$N = 200$				
$\mathbb{E}(L)$	1.95%	1.95%	1.95%	1.95%	0.09%	0.09%	0.09%	0.09%
$VaR_{0.99}(L)$	7.8%	7.26%	48.5%	93.5%	0.9%	0.56%	1.15%	0%
$ES_{0.99}(L)$	9.61%	8.80%	69.20%	99.1%	1.39%	0.78%	1.99%	19.59%
$\mathbb{V}(L)$	0.028%	0.022%	0.74%	1.49%	0.0004%	0.0001%	0.0007%	0.056%
$\mathbb{V}[\mathbb{E}(L \mid M)]$	0.022%	0.022%	0.73%	1.49%	0.0001%	0.0001%	0.0004%	0.056%
$\mathbb{V}^*[\mathbb{E}(L \mid M)]$	80.19%	100%	99.19%	99.86%	33.11%	100%	64.05%	99.69%

Table 11: Risk measures of investment alternatives.

To summarize risk concentrations in tranche portfolios are significantly higher than in normal bond portfolios. Thus, CDO investments implicate that idiosyncratic risk is highly diversified but concentration risk is built up. A more detailed discussion of this topic can be found in Hamerle and Plank [2009].

7 Conclusion

In this article we extensively examined systematic risks with CDOs. We compared risk measures of bonds and CDOs and showed that tranching produces securities with high systematic risk. Furthermore, we investigated drivers of systematic risk with CDOs. On the asset side, increasing the number of names and choosing assets with high systematic risk increases the systematic risk of resulting tranches. In particular CDOs based on collateral pools comprising CDO tranches are very sensitive to systematic factors. On the part of the liability structure we showed that smaller tranches are more systematic risk sensitive. Finally, we shortly addressed the myth of the benefit of CDO diversification. CDO investments are name diversified but contribute significantly to risk factor concentrations.

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